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A STOCHASTIC MULTIRATE SIGNAL PROCESSING APPROACH TO HIGH-RESOLUTION SIGNAL RECONSTRUCTION

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ABSTRACT

The paper addresses the problem of reconstructing a signal at some high sampling rate from a set of signals sampled at a lower rate and subject to additive noise and distortion. A set of periodically time-varying filters are employed in reconstructing the underlying signal. Results are presented for a one-dimensional case involving simulated data, as well as for a two-dimensional case involving real image data where the image is processed by rows. In both cases, considerable improvement is evident after the processing.

1. INTRODUCTION

In many problems of concern to modern signal processing, signals of interest suffer from degradation due to sensor limitation, additive noise, insufficient sampling rate, and various other distortions. High resolution (HR) signal processing, or "super-resolution (SR)" processing as it is frequently called, seeks to produce a more useful signal from a set of observations by exploiting aliasing, subpixel displacement and noise removal in order to produce a higher resolution image.

Work in this area has been spurred on by physical and production limitations on high-precision optical and other imaging systems, as well as, the increased marginal costs associated with achieving greater resolution. In this light, signal processing solutions have become increasingly attractive.

Our approach to this problem is from a stochastic multirate signal processing point of view. Related work includes [1] where an optimal least mean-square estimator is proposed for estimating samples of a random signal based on observations made by several observers and at different sampling rates, and [2] where the problem of fusing two low-rate sensors in the reconstruction of one high resolution signal is considered when time delay of arrival is present. A generalized cross-correlation technique is employed.

In other related work [3, 4, 5, 6] we have investigated optimal linear filtering in estimating an underlying signal from observation sequences at different sampling rates. The focus of these efforts has been on information fusion, i.e., on the combination of observations from multiple sensors to perform tracking, surveillance, classification or some other task. In particular, [3] and [4] considered a simplified problem where an underlying signal was estimated from two sequences, one observed at full rate and the other at half the rate. In [5] least squares formulations were examined where the second sequence had an arbitrary sampling rate. Finally [6] developed a general approach for any number of observation signals at arbitrary sampling rates. In this paper, we apply these methods to the problem of HR signal and image reconstruction. We provide a summary of this method followed by some applications to signal and image processing.

2. PROPOSED METHOD

We consider the problem of estimating a discrete random process $d[n]$, which cannot be observed directly, from a set of M related observation sequences $\{x_0[m_0], x_1[m_1], \ldots, x_{M-1}[m_{M-1}]\}$ related to $d[n]$ through various forms of distortion and interference. These sequences may be sampled at rates lower than that of $d[n]$, and observations sequences at the same rate may also be shifted in time by fractional delays with respect to one another (see e.g. x_0, and x_1 in Fig. 1).

If the desired signal $d[n]$ and its observations $\mathbf{x}_i[m_i]$ are jointly wide-sense stationary, then the linear filters required for optimal mean-square estimation are periodically time-varying [4]. In this case we can write the estimate as

$$\hat{d}_k[n] = \sum_{i=0}^{M-1} \tilde{\mathbf{x}}_i^T \mathbf{h}_i^{(k)} \tag{1}$$

where $\mathbf{h}_i^{(k)}$ is a set of time-varying filter coefficients of length P_i and $\tilde{\mathbf{x}}_i^T$ is a vector of samples from the i^{th} observation sequence. The periodic time variation is denoted

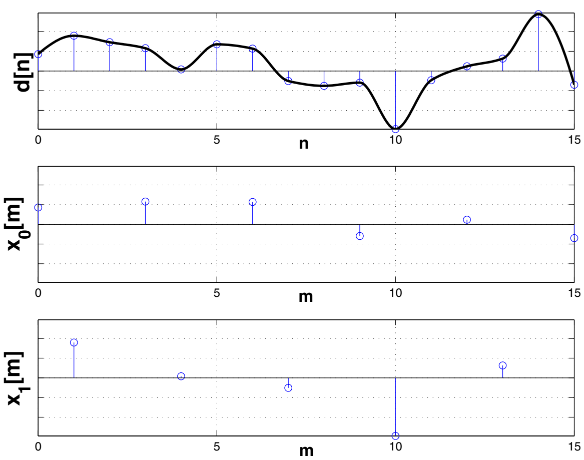


Fig. 1. Observation sequences x_0 and x_1 subsampled ($L = 3$) and shifted by a fractional delay ($k = 0, k = 1$, respectively).

by the index k where $0 \leq k \leq L-1$, L is the system periodicity and $k \equiv n \mod L$ (see Fig. 2).

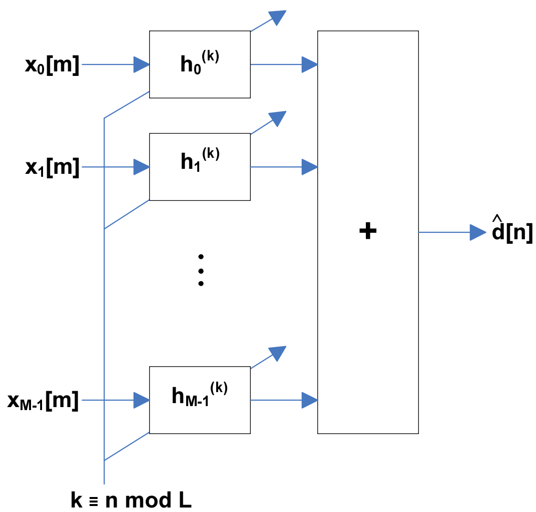


Fig. 2. Reconstruction of the original signal from an ensemble of subsampled signals based on optimal linear filtering

In this paper we consider that the observations signals are maximally decimated versions of $d[n]$ which have been subjected to distortion and additive noise as shown in Fig. 3. In this case the observation vector can be written as

$$\tilde{\mathbf{x}}_i^{(k)} = \mathbf{D}_L^{(k-i)} \tilde{\mathbf{s}}_i[n] \tag{2}$$

and

$$\tilde{\mathbf{s}}_i[n] = [s_i[n], s_i[n-1], \ldots, s_i[n - P_i L + 1]]^T. \tag{3}$$

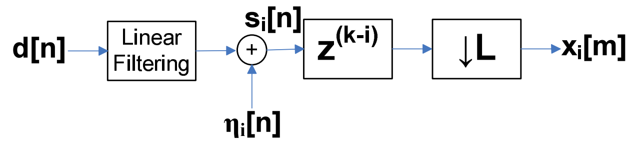


Fig. 3. Observation model for the i^{th} observation sequence.

The matrix $\mathbf{D}_L^{(k)}$ is called a "decimation matrix with time delay" and is used to extract the appropriate samples from $\mathbf{s}_i[n]$ to form each observation vector. The matrix is defined in terms of a Kronecker product of the form

$$\mathbf{D}_L^{(k)} = \mathbf{I} \otimes \boldsymbol{\iota}_k \qquad 0 \leq k \leq L-1 \tag{4}$$

where $\mathbf{I}$ is the $P_i \times P_i$ identity matrix and $\boldsymbol{\iota}_k$ is an $L \times 1$ index vector with a 1 in the $k+1^{th}$ position and 0's elsewhere.

Minimizing the mean-square error [6], leads to a set of Wiener-Hopf equations of the form

$$\begin{bmatrix} \tilde{\mathbf{R}}_{00}^{(k)} & \tilde{\mathbf{R}}_{01}^{(k)} & \ldots & \tilde{\mathbf{R}}_{0L-1}^{(k)} \\ \tilde{\mathbf{R}}_{01}^{(k)*T} & \tilde{\mathbf{R}}_{11}^{(k)} & \ldots & \tilde{\mathbf{R}}_{1L-1}^{(k)} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\mathbf{R}}_{0L-1}^{(k)*T} & \tilde{\mathbf{R}}_{1L-1}^{(k)*T} & \ldots & \tilde{\mathbf{R}}_{L-1L-1}^{(k)} \end{bmatrix} \begin{bmatrix} \mathbf{h}_0^{(k)*} \\ \mathbf{h}_1^{(k)*} \\ \vdots \\ \mathbf{h}_{L-1}^{(k)*} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{r}}_{d0}^{(k)*} \\ \tilde{\mathbf{r}}_{d1}^{(k)*} \\ \vdots \\ \tilde{\mathbf{r}}_{dL-1}^{(k)*} \end{bmatrix}$$
$$0 \leq k \leq L-1 \tag{5}$$

where the time average mean-square error is given by

$$\sigma_\epsilon^2 = R_d[0] - \frac{1}{L} \sum_{k=0}^{L-1} \sum_{j=0}^{L-1} \tilde{\mathbf{r}}_{dj}^{(k)T} \mathbf{h}_j^{(k)} \quad k = 0, 1, \ldots, L-1. \tag{6}$$

The correlation terms are defined as

$$\tilde{\mathbf{r}}_{di}^{(k)} = \mathbf{D}_L^{(k-i)} \tilde{\mathbf{r}}_{di} \tag{7}$$

$$\tilde{\mathbf{R}}_{ij}^{(k)} = \mathbf{D}_L^{(k-i)} \tilde{\mathbf{R}}_{ij} \mathbf{D}_L^{(k-j)*T} \tag{8}$$

and

$$R_d[0] = \mathcal{E}\{d[n] d^*[n]\} \tag{9}$$

where

$$\tilde{\mathbf{r}}_{di} = \mathcal{E}\{d[n] \tilde{\mathbf{s}}_i^*[n]\} \tag{10}$$

and

$$\tilde{\mathbf{R}}_{ij} = \mathcal{E}\{\tilde{\mathbf{s}}_i[n] \tilde{\mathbf{s}}_j^{*T}[n]\}. \tag{11}$$

Solving the multirate Weiner-Hopf equations (5) yields a set of filter coefficients which can be used in the estimation of $d[n]$ as depicted in Figs. 2 and 4.

3. APPLICATION RESULTS

To evaluate the performance of the proposed method two examples are presented. In the first example, a triangular

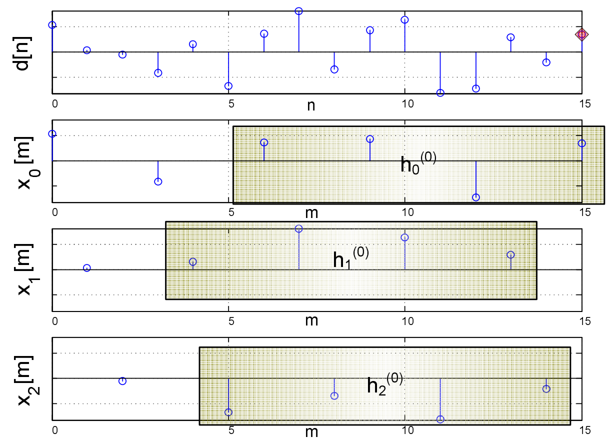
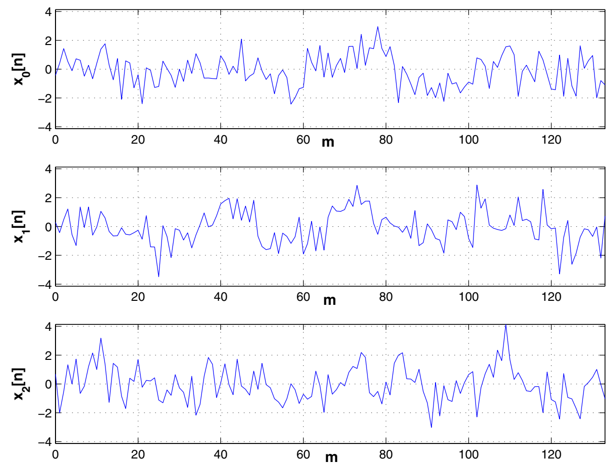
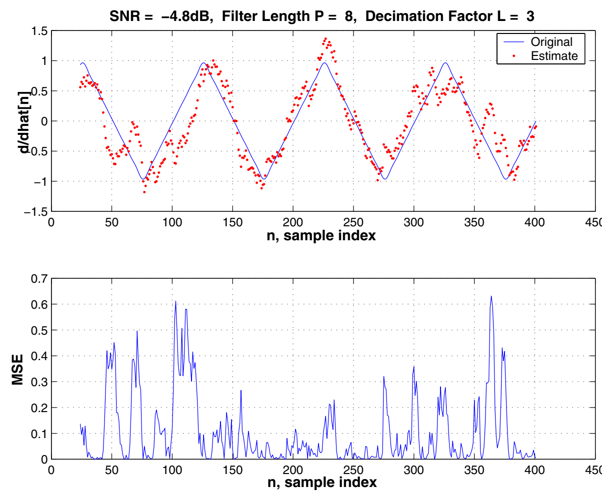


Fig. 4. Reconstruction of the original signal from an ensemble of subsampled signals based on FIR Weiner filtering, decimation factor L = 3, filter order P = 4. The figure illustrates the support of the time-varying filters $\mathbf{h}_i^{(k)}$ at a particular time, $n = 15$ and $k = 0$ (red diamond).



(a) Downsampled, noisy observations, L = 3



(b) Image reconstruction with mean-square error

Fig. 5. Simulation results using optimal linear filtering method for reconstruction, SNR $= -4.8$dB , $P = 8$, $L = 3$.

waveform is considered for reconstruction. Our method was compared to the method described in [7] which can produce an exact reconstruction of the triangular waveform if the highest frequency terms are left out. Both methods produce accurate results when there is no noise added to the observation sequences. When a small amount of noise is added to the observation sequences the exact reconstruction method fails to reliably reproduce the signal, while the method described here continues to produce a reasonably good approximation to the signal even under severely noisy conditions (see Fig. 5).

Three observation sequences are given as depicted in (a) after being subjected to additive white gaussian noise. The sequences are subsampled by a factor of $L = 3$. Note that the underlying form of the original sequence is undetectable. After processing, the HR triangular waveform (unobserved) is reconstructed from these LR observation sequences and is depicted in (b). It is compared to the original sequence and its mean-square error is displayed. Note the close correspondence between the estimate and original in a relatively low SNR environment.

As a second example we apply this method to the problem of SR image reconstruction. We consider each row of the observed LR images as an observation signal vector belonging to the set $\{\mathbf{x}_0[m], \mathbf{x}_1[m], \ldots, \mathbf{x}_{M-1}[m]\}$ (Fig. 6). Reconstruction is then accomplished line-by-line until every row of each image is processed. In this case, the original image is depicted in (a) and one of its three subsampled observation images with additive white gaussian noise is given in (b). The image depicted in (c) represents the result of applying standard nearest-neighbor interpolation to one of the three noisy subsampled images and image (d) is the reconstructed image. Although the image is processed in only one direction, there is significant improvement over the interpolated image. In particular, note that edges of structures can be observed in many cases where the interpolated image does not provide such detail.

4. CONCLUSIONS

In this paper an optimal linear filtering approach is introduced for the HR reconstruction of an unobserved random process from a set of related subsampled processes. This method involves optimal periodically time-varying filters that are applied to the data. The success of this method was demonstrated in the reconstruction of a known signal (triangular waveform) from noisy, low-rate observations and in

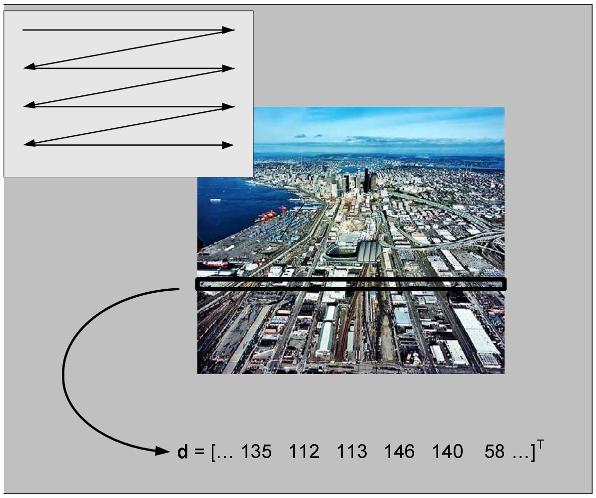


Fig. 6. Line-by-line processing of observation images.

reconstruction of a HR image from noisy LR versions by processing in a row-wise manner. The success in reconstructing the original image motivates future work which will consist of further extending these methods to two- and three-dimensional data.

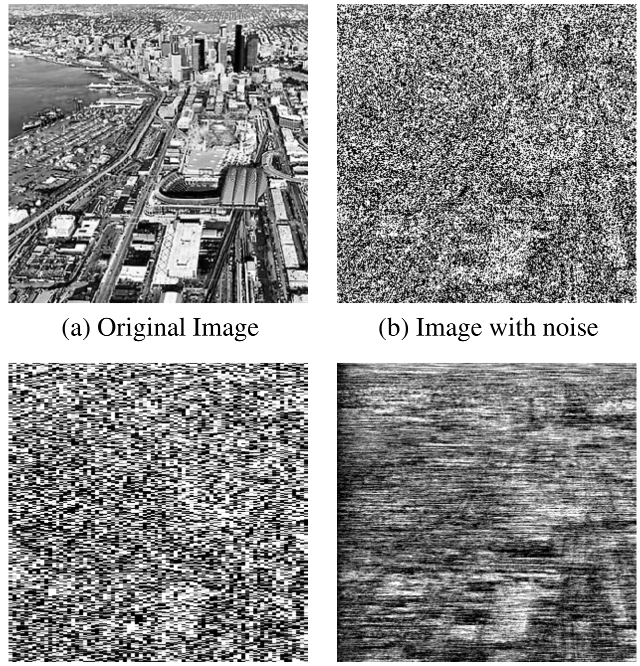


Fig. 7. Simulation results comparing optimal linear filtering method to nearest-neighbor interpolation.

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